Joint Scheduling and Wireless Network Coding

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***Abstract*— In this paper, we establish a new framework for network coding in ad hoc wireless networks. First, we consider a simple wireless network topology to illustrate how network coding can improve throughput and energy efficiency objectives beyond routing solutions. Then, we extend the network coding problem to general wireless networks in conjunction with scheduling-based medium access control (MAC). For that purpose, we partition the nodes into disjoint sets of transmitters and receivers that result in conflict-free network realizations with minimum cost (e.g. power) assignments. We separately activate distinct network realizations using a time division mechanism. Then, we specify the content of network flows through network coding and derive transmission schedules to optimize the throughput or energy measures. Next, we present a method of constructing time-varying linear network codes that satisfy the wireless network properties. Also, we verify via numerical results the superior performance of network coding over routing in terms of throughput and energy efficiency.**

I. Introduction

It is known that coding over networks enables connections with rates that are superior to those achieved through classical routing [1]. In routing, nodes simply replicate and forward the received packets, whereas network coding allows relay nodes to combine the information received from multiple links for subsequent transmissions. As shown in [2]-[3], linear network coding is sufficient to achieve the maximum flow bounds between the source-destination pairs in wired networks.

To extend network coding to a wireless packet network, we need to impose additional properties such as (P1) omnidirectional transmissions, (P2) single packet transmission or reception by any node at any given time, (P3) no simultaneous packet transmission and reception by any node (due to the constraint that nodes are equipped with a single transceiver), (P4) possible destructive interference effects among concurrent transmissions, and (P5) multihop packet propagation in store-and-forward manner instead of continuous information flow. These wireless communication properties introduce new cross-layer interactions between MAC and network coding that have not been addressed in the context of wired network coding.

Network coding has been extended in [4] to randomized (but non-wireless) environments with distributed implementation. Network coding in the presence of omnidirectional transmissions has been studied in [5] through the use of linear programs to optimize network resources based on link costs. As a special case, network coding has been addressed in [6] for energy-efficient multicasting. In this paper, our objective is to incorporate realistic interference and delay effects and practical constraints of a single transceiver per node. In addition, we consider dynamic operation of multihop packet communication in contrast to previous models of connection-oriented traffic. We also model the performance measures such as throughput and energy consumption in terms of node costs.

We analyze wireless network coding in conjunction with scheduling-based MAC and propose a two-step solution: 1. We predetermine a finite set of feasible (conflict-free) wireless network realizations, and assign minimum costs (e.g. power) to each node for any network realization, 2. We assign time fractions to network realizations and choose the flows between transmitter-receiver pairs (originating at different sources and addressed to different destinations) to optimize the performance measures (e.g. throughput or energy cost) through network coding. We specify the properties of linear network codes for wireless networks and present a method of constructing these codes to achieve the resulting network flows. Then, we present numerical results to evaluate the throughput and energy efficiency properties of network coding and routing solutions.

II. The Model of Wireless Ad Hoc Network

We assume omnidirectional transmissions that are synchronized into unit time slots. Each node is equipped with a single transceiver and cannot simultaneously transmit and receive packets. Therefore, it is necessary to partition the nodes into disjoint transmitter and receiver sets at any time slot. We do not allow multiple packet transmissions or receptions by any node in a single slot. We consider two different channel models:

1. Classical collision channel model: We assume circular transmission (reception) ranges with sharp boundaries. No successful transmission or interference is possible beyond those ranges. We model each link as a collision channel with three possible channel outcomes: idle, success, and collision that occur, respectively, if none, one or more than one packets simultaneously reach the same receiver in the same time slot.

2. Physical channel model: A transmission from node i to node j is successfully received at time slot t if and only if

$$\frac{P_i(t) g_{i,j}}{\sum_{k \in V \setminus \{i,j\}} P_k(t) g_{k,j} + \eta_j} \geq \gamma \tag{1}$$

where $P_i(t)$ is the power of node i at time slot t, $g_{i,j}$ is the channel gain from node i to j, η_j is the additive Gaussian noise power at receiver j, γ is the common Signal-to-Interference-plus-Noise-Ratio (SINR) threshold, and V is the set of nodes.

III. An Example of Wireless Network Coding

We consider the problem of multicasting in the wireless network shown in Figure 1-(A) to illustrate the advantages

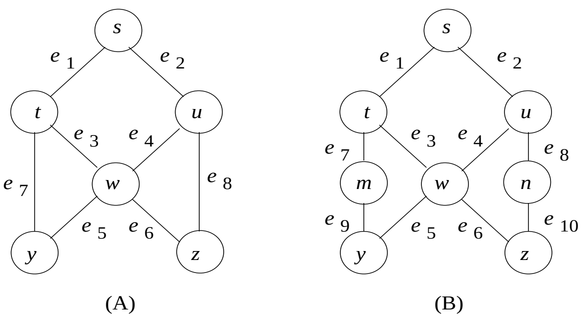

Fig. 1. Wireless network topologies (A) and (B)

of network coding over routing. We compare two different multihop communication strategies: (1) classical routing that limits nodes to act as forwarding switches, (2) network coding that allows nodes to code over the received information.

The objective of the wireless multicasting problem is to deliver the packets of the source node s to both destination nodes y and z. A packet transmission reaches all nodes connected through a single link (as shown in Figure 1-(A)) to the transmitter node. If multiple transmissions reach a node in the same time slot, a packet collision occurs. We consider conflict-free transmission scheduling under the classical collision channel model. For simplicity of the illustration here, we assume that each packet contains one bit and the source s has always a packet to transmit. Each transmission consumes an amount of $\mathcal{E}$ energy units. We consider the following performance objectives: (1) r: throughput per destination (average number of packets delivered to each destination per unit time), (2) $\mathcal{E}_{avg}$: average energy consumed to deliver a packet to any destination, and (3) $\mathcal{D}_{avg}$: average delay per packet. We denote by $c\,\overrightarrow{b}\,d$ the transmission of bit b from node c to node d. The throughput and energy-optimal routing solution schedules transmissions $s\,\overrightarrow{b}\,\{t,u\}$ in odd time slots and $t\,\overrightarrow{b}\,y$, $u\,\overrightarrow{b}\,z$ in even time slots for bit b. This routing solution achieves $r = \frac{1}{2}$ bits/slot, $\mathcal{D}_{avg} = 2$ slots and $\mathcal{E}_{avg} = \frac{3}{2}\mathcal{E}$ energy units per bit.

The conflict-free transmissions (with period of three slots after the initial slot) for optimal network coding are given in Table I. The linear network coding operation consists of node w performing the bit addition $b_{2k-1} + b_{2k}$ at slots $3k+1$, $k = 1, 2, \ldots$, and sending the bit sum to nodes y and z in a single transmission. Since bits b_{2k-1} and b_{2k} have been delivered to nodes y and z in the previous transmissions, nodes y and z combine b_{2k-1} and b_{2k} with $b_{2k-1} + b_{2k}$, respectively, to recover both b_{2k-1} and b_{2k}. As time evolves, r approaches $\frac{2}{3}$ bits/slot, $\mathcal{D}_{avg}$ approaches $\frac{13}{4}$ slots and $\mathcal{E}_{avg}$ approaches $\frac{5}{4}\mathcal{E}$ energy units per bit. Network coding requires node w to store 2 bits, whereas the buffer size of one is sufficient for routing.

Consider the network shown in Figure 1-(B). The transmission schedule for the best routing solution (in terms of throughput and energy consumption) is given by $s\,\overrightarrow{b_{k+1}}\,t$ (or u) at time slot $3k+1$, t (or u) $\overrightarrow{b_{k+1}}\,w$ at time slot $3k+2$, and $w\,\overrightarrow{b_{k+1}}\,\{y,z\}$ at time slot $3k+3$ for any non-negative integer k and bit b_{k+1}. This routing solution achieves $r = \frac{1}{3}$ bits/slot, $\mathcal{D}_{avg} = 3$ slots and $\mathcal{E}_{avg} = \frac{3}{2}\mathcal{E}$ energy units per bit. The network coding solution given in Table II extends r to $\frac{1}{2}$ bits/slot, whereas $\mathcal{E}_{avg}$ and $\mathcal{D}_{avg}$ increase to $\frac{7}{4}\mathcal{E}$ energy units per bit and 5 slots, respectively. Thus, the throughput and energy efficiency objectives can conflict with each other. This observation suggests the need for joint optimization of the performance measures such as throughput and energy costs.

TABLE I

NETWORK CODING SOLUTION FOR THE NETWORK IN FIGURE 1-(A)

Time Slot	1	2	3	4
Strategy	$s\,\overrightarrow{b_1}\,t$	$t\,\overrightarrow{b_1}\{w,y\}$ $s\,\overrightarrow{b_2}\,u$	$u\,\overrightarrow{b_2}\,\{w,z\}$ $s\,\overrightarrow{b_3}\,t$	$w\,\overrightarrow{b_1+b_2}\,y$ $w\,\overrightarrow{b_1+b_2}\,z$
Time Slot	5	6	7	8
Strategy	$t\,\overrightarrow{b_3}\,\{w,y\}$ $s\,\overrightarrow{b_4}\,u$	$u\,\overrightarrow{b_4}\,\{w,z\}$ $s\,\overrightarrow{b_5}\,t$	$w\,\overrightarrow{b_3+b_4}\,y$ $w\,\overrightarrow{b_3+b_4}\,z$	$t\,\overrightarrow{b_5}\,\{w,y\}$ $s\,\overrightarrow{b_6}\,u$

TABLE II

NETWORK CODING SOLUTION FOR THE NETWORK IN FIGURE 1-(B)

Time Slot	1	2	3	4
Strategy	$s\,\overrightarrow{b_1}\,t$	$t\,\overrightarrow{b_1}\,\{w,m\}$ $s\,\overrightarrow{b_2}\,u$	$u\,\overrightarrow{b_2}\,\{w,n\}$ $s\,\overrightarrow{b_3}\,t$	$m\,\overrightarrow{b_1}\,y$ $n\,\overrightarrow{b_2}\,z$
Time Slot	5	6	7	8
Strategy	$w\,\overrightarrow{b_1+b_2}\,y$ $w\,\overrightarrow{b_1+b_2}\,z$	$t\,\overrightarrow{b_3}\,\{w,m\}$ $s\,\overrightarrow{b_4}\,u$	$u\,\overrightarrow{b_4}\,\{w,n\}$ $s\,\overrightarrow{b_5}\,t$	$m\,\overrightarrow{b_3}\,y$ $n\,\overrightarrow{b_4}\,z$

IV. GENERAL PROBLEM OF WIRELESS NETWORK CODING

We consider a network graph $G = (V, E)$, where V is the set of nodes and E is the set of links. We impose the wireless communication properties introduced in sections I-II. A single source s wishes to send traffic with common rate r to each destination in D. The information packets cannot continuously flow in multihop wireless networks, since (a) packets travel at most one hop per unit time, (b) simultaneous transmissions over different links may not be feasible due to interference effects, (c) nodes can either transmit or receive at any given time. Therefore, we need to define time-varying network flows. The total flow on link (i,j) at time slot t is $z_{i,j}(t)$. The fraction of $z_{i,j}(t)$ destined to node $d \in D$ is $x_{i,j}(d,t)$. We assume that node i incurs energy cost $P_i(t)$ at time slot t. Our formulation involves (as it should) only node-based costs (e.g. transmission power) rather than link-based costs as is done in wired networks [5]. For any $d \in D$, $i \in V$, $j \in V$ and time slot t, the conditions on wireless network flows are:

$$x_{i,j}(d,t) \text{ and } z_{i,j}(t) \in \{0,1\}, x_{i,j}(d,t) \leq z_{i,j}(t) \quad (2)$$

(a) Classical Collision Channel: $z_{i,j}(t) = 1$ if

$$P_i(t) \geq P_{i,j}, P_k(t) < P_{k,j}\ \forall k \neq i \text{ and } P_j(t) = 0 \quad (3)$$

(b) Physical Channel: $z_{i,j}(t) = 1$ if

$$\frac{P_i(t) g_{i,j}}{\sum_{k \in V \setminus \{i,j\}} P_k(t) g_{k,j} + \eta_j} \geq \gamma \text{ and } P_j(t) = 0 \quad (4)$$

$$\lim_{K \to \infty} \frac{1}{K} \sum_{t=1}^{K} \Big(\sum_{j \in V \setminus \{i\}} x_{i,j}(d,t) - \sum_{j \in V \setminus \{i\}} x_{j,i}(d,t) \Big)$$
$$= r \text{ if } i = s, \quad -r \text{ if } i \in D, \quad \text{or } 0 \text{ otherwise} \quad (5)$$

where $P_{i,j}$ is the transmission power required by node i to reach node j over a single hop in a classical collision channel.

If $P_{i,j} = \infty$, then there is no direct link possible from node i to node j. We assume zero cost for packet reception. Our analysis, however, can be extended to the case where there is also an energy cost for reception. The two-step solution to the problem of joint MAC and network coding is given as follows:

Step 1: Assign a transmission power cost $P_i(t)$ to each node $i \in V$ at any time slot t. This uniquely determines the flows $z_{i,j}(t)$ from i to any other node j according to the condition (3) or (4) depending on which channel model we use. The cost for any idle or receiver node i at time slot t is $P_i(t) = 0$ such that $z_{i,j}(t) = 0$ for any other node j. Hence, the cost assignment also results into a partitioning of the nodes into the disjoint sets of transmitters and receivers.

Step 2: Choose the flows $x_{i,j}(d,t)$ at each time slot t (subject to conditions (2) and (5)) through network coding (or routing as a special case) in order to either (I) maximize the stable throughput per destination r, or (II) minimize the average cost per time slot $a = \lim_{K\to\infty} \frac{1}{K}\sum_{t=1}^{K} a(t)$ for fixed r, where $a(t) = \sum_{i\in V} P_i(t)$, or (III) minimize the energy cost per successfully delivered packet (or equivalently minimize $\frac{a}{r}$).

V. Joint Scheduling and Network Coding

The resulting problem from this formulation is rather complex, since it involves non-linear optimization at each time slot. Instead, we propose a periodic solution with reduced complexity. We periodically activate distinct network realizations (i.e. conflict-free link sets) each of which is allocated in a non-overlapping time interval. Then, we perform the optimization individually for each network realization rather than for each time slot. We define $N^f = \{N^f_m, m=1,...,M\}$ as the set of feasible network realizations. The network realization N^f_m (with node set V^f_m and link set E^f_m) is allocated in time interval $\mathcal{T}_m$ over τ_m fraction of the total time such that $P_i(t) = P_i^{(m)}$, $z_{i,j}(t) = z_{i,j}^{(m)}$ and $x_{i,j}(t,d) = x_{i,j}^{(m)}(d)$ for all $t \in \mathcal{T}_m$. The network realizations yield conflict-free transmissions on activated links and partition the network into disjoint transmitter and receiver sets at any time slot. For any $d \in D$, $i \in V$, $j \in V$ and $m = 1,...,M$, we have the following flow conditions:

$$x_{i,j}^{(m)}(d) \text{ and } z_{i,j}^{(m)} \in \{0,1\}, x_{i,j}^{(m)}(d) \le z_{i,j}^{(m)} \tag{6}$$

(a) Classical Collision Channel: $(i,j) \in E^f_m, z_{i,j}^{(m)} = 1$ if

$$P_i^{(m)} \ge P_{i,j}, P_k^{(m)} < P_{k,j}\ \forall k \neq i \text{ and } P_j^{(m)} = 0 \tag{7}$$

(b) Physical Channel: $(i,j) \in E^f_m, z_{i,j}^{(m)} = 1$ if

$$\frac{P_i^{(m)} g_{i,j}}{\sum_{k\in V\setminus\{i,j\}} P_k^{(m)} g_{k,j} + \eta_j} \ge \gamma \text{ and } P_j^{(m)} = 0 \tag{8}$$

$$\sum_{m=1}^{M} \tau_m \left(\sum_{j:(i,j)\in E^f_m} x_{i,j}^{(m)}(d) - \sum_{j:(j,i)\in E^f_m} x_{j,i}^{(m)}(d) \right)$$
$$= r \text{ if } i = s, \quad -r \text{ if } i \in D, \quad \text{or } 0 \text{ otherwise} \tag{9}$$

We propose the following two-step solution to the problem of joint scheduling and network coding:

Step 1: Predetermine the network realizations N^f with the minimum transmission power assignments that ensure conflict-free transmissions. The flow $z_{i,j}^{(m)}$ for any i and j is uniquely determined by N^f_m and $\{P_i^{(m)}\}_{i\in V}$ under condition (7) or (8).

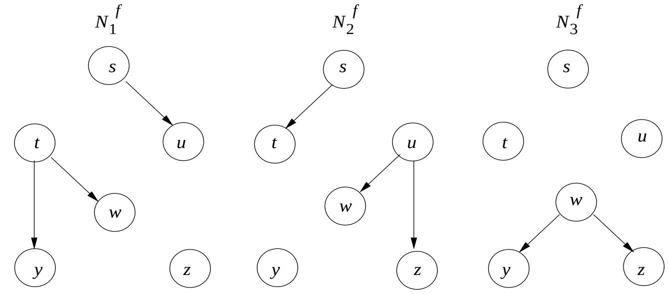

Fig. 2. Network realizations $N^f = \{N^f_1, N^f_2, N^f_3\}$

Step 2: Assign the time fractions $\{\tau_m\}_{m=1}^{M}$ to network realizations $\{N^f_m\}_{m=1}^{M}$ and choose the flows $x_{i,j}^{(m)}(d)$ (under conditions (6) and (9)) through network coding (or routing) in order to (I) maximize r, or (II) minimize $a = \sum_{m=1}^{M} \tau_m a^{(m)}$ for fixed r, where $a^{(m)} = \sum_{i\in V} P_i^{(m)}$, or (III) minimize $\frac{a}{r}$.

The optimal solution requires extensive search over all possible network realizations and does not scale with increasing number of nodes. This is the classical combinatorial optimization problem encountered under scheduling. Therefore, we use the following heuristic to determine the network realizations.

A. Heuristic to Construct Wireless Network Realizations

We start by constructing the first network realization; we choose a node arbitrarily as receiver, and designate as its transmitter the node with the largest channel gain from the chosen receiver under the physical channel model or the node with the smallest power to reach the chosen receiver under the classical collision channel model. We choose as the second receiver an, until now, unchosen node, and designate as its transmitter a node that has not been previously activated as receiver and has the largest channel gain or the smallest power requirement depending on the channel model. We admit this transmitter-receiver pair, if the activation of this link does not destructively interfere with previously admitted transmissions.

Under the physical channel model, we run a power control algorithm that determines the minimum power for the transmitter to reach the intended receivers without destroying the transmissions already admitted. Under the classical collision channel model, we check whether the chosen transmitter has a non-intended receiver in its transmission range. If so, then we choose another transmitter and run the same admissibility check algorithm. We proceed in this fashion and determine transmitter-receiver pairs until no link can be admitted without distorting the already admitted conflict-free link assignments.

Subsequently, we repeat the same procedure by choosing as receiver a node previously designated as transmitter and running the same algorithm to determine the complete set of $N^f = \{N^f_m\}_{m=1}^{M}$, until each node is designated as transmitter and receiver at least once (with the exception of the source node that should be only activated as transmitter). Each realization N^f_m partitions the nodes into the disjoint transmitter and receiver sets $T^{(m)}$ and $R^{(m)}$, respectively. An example of wireless network realizations for the network in Figure 1-(A) is shown in Figure 2. Under the classical collision channel model, the link set E^f_m is conflict-free, if the condition (7)

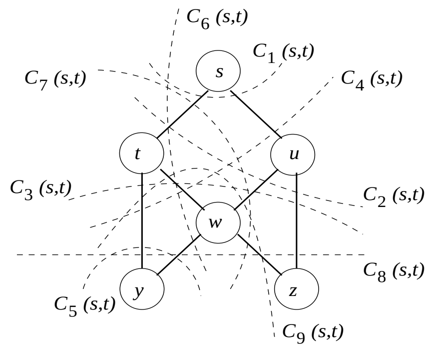

Fig. 3. Example of cuts between nodes s and y of network in Figure 1-(A)

holds for all flows on links $(i,j) \in E_m^f$. The cost of node i for the network realization N_m^f is $P_i^{(m)} = \max_{j:(i,j)\in E_m^f} P_{i,j}$.

For the SINR-based physical channel model, we define $R^{(m)}(i)$ as the receiver set for the transmitter $i \in T^{(m)}$ that has the transmission power $P_i^{(m)}$ for the network realization N_m^f. We schedule transmissions to any receiver only from the transmitter node with the largest channel gain to that receiver, as implemented by the heuristic of section V-A. We define the matrix $H^{(m)}$ such that $(H^{(m)})_{i,j} = h_{i,j}^{(m)}$ if $i \neq j$, and 0 if $i = j$, where $h_{i,j}^{(m)} = \max_{k\in R^{(m)}(i)} \frac{g_{j,k}}{g_{i,k}}$. We define $\underline{P}^{(m)}$ and $\underline{\sigma}$ as the power and noise vector such that $(\underline{P}^{(m)})_i = P_i^{(m)}$ and $(\underline{\sigma})_i = \sigma_i$, $i \in T^{(m)}$, where $\sigma_i = \max_{k\in R^{(m)}(i)} \frac{\eta_k}{g_{i,k}}$. The SINR condition for the activated transmitters can be expressed as

$$\underline{P}^{(m)} \geq \gamma \, (H^{(m)}\underline{P}^{(m)} + \underline{\sigma}) \tag{10}$$

for each network realization N_m^f. Theorem 1 that follows is an extension of the one-to-one scheduling result from [7].

Theorem 1: (a) $H^{(m)}$ is an irreducible non-negative matrix.

(b) If the Perron-Frobenius eigenvalue of $H^{(m)}$ is strictly smaller than $\frac{1}{\gamma}$, the wireless network realization N_m^f is conflict-free and the optimal solution that minimizes $\sum_{i\in T^{(m)}} P_i^{(m)}$ is given by $\underline{P}^{(m)*} = \gamma \, (I - \gamma H^{(m)})^{-1} \, \underline{\sigma}$.

(c) A distributed solution that converges to $\underline{P}^{(m)*}$ is $\underline{P}^{(m)}(l+1) = I^{(m)}(\underline{P}^{(m)}(l))$, where $P^{(m)}(l)$ is the power vector at the lth iteration and $I^{(m)}(\underline{P}) = \gamma \, (H^{(m)}\underline{P}+\underline{\sigma})$ is a standard interference function, as defined in [7].

B. Time Allocation to Wireless Network Realizations

We define the kth cut between nodes s and d as $C_k(s,d)$. For wired networks, the value of a cut is the sum of the capacities of the links that cross the given cut. For wireless networks, we introduce the average cut value $c_k^{N^f}(s,d)$, which is the maximum number of successful transmissions (time-averaged over all network realizations in N^f) across the cut $C_k(s,d)$ per unit time. To incorporate omnidirectional transmissions, the contribution of a node to any cut is limited to at most 1 per unit time, since a single node can transmit at most one distinct information (packet) over each cut at any time slot. For the network realizations N^f in Figure 2, Figure 3 depicts an example of the cuts between nodes s and y with the cut values: $c_1^{N^f}(s,y) = \tau_1 + \tau_2$, $c_2^{N^f}(s,y) = 2\tau_2$, $c_3^{N^f}(s,y) = \tau_1 + \tau_2$, $c_4^{N^f}(s,y) = 2\tau_1$, $c_5^{N^f}(s,y) = \tau_1 + \tau_3$, $c_6^{N^f}(s,y) = \tau_2 + \tau_3$, $c_7^{N^f}(s,y) = 2\tau_2$, $c_8^{N^f}(s,y) = \tau_1 + \tau_2 + \tau_3$ and $c_9^{N^f}(s,y) = \tau_1 + \tau_2$. The maximum flow from the source s to destinations is $\max_{\{\tau_m,\, m=1,\ldots,M\}}\{\min_{d\in D} \min_k c_k^{N^f}(s,d)\}$. For the network realizations N^f in Figure 2, the maximum flow from the source s to any destination is $\min(2\tau_1, 1-\tau_1)$ for $\tau_1{=}\tau_2$, and it is maximized by $\tau_m{=}\frac{1}{3}$, m=1,2,3. This is equivalent to the network coding solution in Table I. In section VII, we will derive the wireless network codes that achieve the resulting maximum flows. For the network in Figure 1-(A), we also consider the alternative problems of selecting $\{\tau_m\}_{m=1}^M$ in order to minimize a for fixed r or minimize $\mathcal{E}_{avg}$.

(a) Classical collision channel: Assume unit energy cost for any transmission. The energy-optimal network coding solution uses the network realizations in Figure 2 with $a^{(1)}{=}a^{(2)}{=}2$, $a^{(3)}{=}1$, and achieves the average cost $a{=}1{+}2\tau_1$ for $r{=}\min(2\tau_1, 1{-}\tau_1)$. The assignment of $\tau_m{=}\frac{1}{3}$, m=1,2,3, minimizes $\mathcal{E}_{avg}{=}\frac{a}{2r}$ to $\frac{5}{4}$ energy units per packet while maximizing r to $\frac{2}{3}$ packets per slot. The minimum value of $\mathcal{E}_{avg}$ for the best routing solution is $\frac{3}{2}$ energy units per packet.

(b) Physical channel: Assume that the distance between any connected node pair (i,j) in Figure 1-(A) is $d(i,j){=}1$, $g_{i,j}{=}1/d(i,j)^2$ and $\eta_j{=}1$ for $j{\in}V$. If we consider the network realizations in Figure 2 for throughput-optimal network coding, the non-zero power costs are $P_s^{(1)}{=}P_s^{(2)}{=}\frac{\gamma\,(1+\gamma/3)}{1-\gamma^2/3}$, $P_t^{(1)}{=}P_u^{(2)}{=}\frac{\gamma\,(1+\gamma)}{1-\gamma^2/3}$ and $P_w^{(3)}{=}\gamma$, where $\gamma{<}\sqrt{3}$. The best routing solution assigns the non-zero power costs of $P_s^{(1)}{=}\gamma$, $P_t^{(2)}{=}P_u^{(2)}{=}\frac{\gamma}{1-\gamma/4}$, where $\gamma{<}4$. If we have $\gamma{>}\ 0.4291$, this routing solution is equivalent to the energy-optimal network coding solution and achieves smaller values of $\mathcal{E}_{avg}$ compared to the case of throughput-optimal network coding.

VI. Properties of Linear Wireless Network Codes

In wireless networks, nodes encode and transmit packets or receive and decode packets at different time instants. Hence, we need time-varying network coding that distinguishes when information is generated or received and when information is encoded or decoded at any node. In this section, we extend linear network coding [2]-[3] to wireless packet networks. The flow $Y_{e,t+1}$ on link $e \in E$ at time $t+1$ is encoded as

$$Y_{e,t+1} = \sum_{l=t-p}^{t} \sum_{e'\in E:\, head(e')=tail(e)} \beta_{e',e,l,t+1} Y_{e',l} + \sum_{l=t-p}^{t} \sum_{j=1}^{\mu(tail(e))} \alpha_{j,e,l,t+1} X_l(tail(e), j) \tag{11}$$

where $head(e){=}j$, $tail(e){=}i$ for $e{=}(i,j)$ and p is the length of coding memory. The flow $Y_{e',l}$ that arrives on $e' \in E$ at time $l \leq t$ is mapped by $\beta_{e',e,l,t+1}$ onto $e \in E$ at time $t+1$. The input flow $X_l(tail(e), j)$ generated by node $tail(e)$ at time $l \leq t$ is mapped by $\alpha_{j,e,l,t+1}$ onto $e \in E$ at time $t+1$. The kth output flow decoded by node $v \in V$ at time t is

$$Z_t(v,k) = \sum_{l=t-p}^{t} \sum_{e'\in E:\, head(e')=v} \epsilon_{e',k,l,t} Y_{e',l} \tag{12}$$

where $\epsilon_{e',k,l,t}$ maps the incoming flow $Y_{e',l}$ on $e' \in E$ at time $l \leq t$ to reconstruct the kth flow at time t. We have the conditions $\beta_{e',e,l,t+1}=0$, $\alpha_{j,e,l,t+1}=0$, $\epsilon_{e,k,l,t}=0$, if $l>t$ or $head(e')\neq tail(e)$. To minimize the packet delay, we assume that any packet that will be transmitted at time slot $t+1$ is generated at time slot t, i.e. $\alpha_{j,e,l,t+1}=0$ for all $l\neq t$.

Theorem 2: If we impose omnidirectional transmissions, we have the following properties for linear network encoding:

$$\beta_{e',e,l,t}=\beta_{e',l,t} \text{ for } e\in E: tail(e)=head(e') \quad (13)$$

$$\alpha_{j,e,l,t}=\alpha_{j,l,t}(tail(e)) \text{ for } e\in E \quad (14)$$

Proof: All links out of a single node carry the same information at any time slot t, i.e. we have

$$Y_{e,t}=Y_t(tail(e)) \text{ for } e\in E \quad (15)$$

If we impose condition (15), the encoding coefficients must satisfy (13)-(14) so that linear encoding (11) can hold. ■

Thus, we are better served by using node-based (rather than link-based) network encoding for wireless networks.

In addition to omnidirectional transmissions, we assume that there is only a single transceiver per node. Therefore, nodes cannot transmit and receive packets at the same time slot. For simplicity, we consider the finite field F_2 for linear encoding and decoding operations. Similar extensions to arbitrary operation fields are possible but omitted for brevity.

Theorem 3: Assume omnidirectional transmissions and single transceiver per node. Consider conflict-free scheduling that partitions the nodes into the disjoint sets of transmitters $T(t)$ and receivers $R(t)$ at any time slot t. The linear network coding operations must satisfy the following properties:

$$\beta_{e,l,t}=0 \text{ and } \alpha_{j,l,t}(v)=0 \text{ if } v=head(e)\in R(t) \quad (16)$$

$$\exists(e,l,j) \text{ s.t. } \beta_{e,l,t}=1 \text{ or } \alpha_{j,l,t}(head(e))=1, \text{ and} \quad (17)$$

$$\epsilon_{e,j,l,t}=\epsilon_{e,j,l,t-1}, \epsilon_{e,j,t,k}=\beta_{e,t,k}=0 \text{ if } head(e)\in T(t)$$

Proof: If node v is activated as receiver at time t, i.e. $v{\in}R(t)$, it cannot transmit at time t any information that has been received or generated at time $l{<}t$, i.e. $\alpha_{j,l,t}(v){=}0$ and $\beta_{e,l,t}{=}0$ if $head(e){=}v$. The conditions for receiver nodes are given by (16). Node v is activated as transmitter, only if there is at least one node that can successfully receive the packets of v. Hence, node $v\in T(t)$ uses at least one non-zero encoding coefficient at time t. Node $v\in T(t)$ does not receive any information to be transmitted later at time $k{>}t$, i.e. $\beta_{e,t,k}{=}0$ for $head(e){=}v$. Also, there is no need to change the decoding coefficients of $v{\in}T(t)$, i.e. $\epsilon_{e,j,l,t}{=}\epsilon_{e,j,l,t-1}$ for $head(e){=}v$, and there is no new information that can be decoded to any flow j at time $k{>}t$, i.e. $\epsilon_{e,j,t,k}{=}0$ for $head(e){=}v$. The conditions for transmitter nodes are given by (17). ■

If we assume periodic operation over predetermined network realizations, we can replace the time indices in network coding coefficients with the indices of network realizations.

VII. Construction of Wireless Network Codes

Consider the predetermined wireless network realizations $N^f=\{N^f_m\}_{m=1}^M$ with time allocations $\{\tau_m\}_{m=1}^M$. We define a hypothetical connected (wired) network $N^g=(V^g,E^g)$ with the node set $V^g=\bigcup_{m=1}^M V^f_m$ and link set $E^g=\bigcup_{m=1}^M E^f_m$. The capacity of any link $(i,j)\in E^g$ out of node $i\in V^g$ is $c_i{=}\sum_{m=1}^M \tau_m\, 1\,(i\in T^{(m)})$, where 1 is the indicator function.

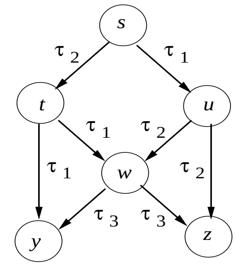

Fig. 4. The hypothetical wired network graph N^g

Figure 4 depicts N^g for the network realizations in Figure 2. To incorporate omnidirectional transmissions while keeping the original definition of cuts and flows, we can connect an artificial node to each transmitting node with an error-free link of capacity 1 (as proposed by [6]) such that the maximum flow over at least one cut has the value of 1 for a transmitting node. Instead, we propose a new formulation that assigns the value of at most 1 (per unit time) to a cut that crosses (possibly) multiple links out of a single node in both networks N^f and N^g. This approach is preferable, since it allows us to associate node-based costs (e.g. power) with each transmission.

Lemma 1: The network realizations N^f and network graph N^g have the same cut values and the same maximum flows.

Proof: We define $c_k^{N^g}(s,d)$ and $c_k^{N^f}(s,d)$ as the values of the cut $C_k(s,d)$ for the wired network graph N^g and for the wireless network realizations $N^f=\{N^f_m\}_{m=1}^M$ (with time allocations $\{\tau_m\}_{m=1}^M$), respectively. Since we assign the value of 1 to a cut that has multiple links out of a single node, we define $c_k^{N^g}(s,d)=\sum_{i\in V^g} 1(i{\to}C_k(s,d))\cdot c_i$ and $c_k^{N^f}(s,d)=\sum_{m=1}^M \tau_m \sum_{i\in T^{(m)}} 1(i{\to}C_k(s,d))$, where $i{\to}C_k(s,d)$ means that at least one link out of node i crosses the cut $C_k(s,d)$. Since $c_i=\sum_{m=1}^M \tau_m\, 1\,(i\in T^{(m)})$ and $T^{(m)}\subseteq V^g$, we obtain $c_k^{N^g}(s,d)=\sum_{i\in V^g} 1\,(i\to C_k(s,d))\sum_{m=1}^M \tau_m\, 1\,(i\in T^{(m)})$ $=\sum_{m=1}^M \tau_m \sum_{i\in T^{(m)}} 1(i\to C_k(s,d))=c_k^{N^f}(s,d)$. Thus, N^f and N^g have the same cut values. According to the maximum-flow/minimum-cut theorem, the maximum flow between nodes s and d is $\min_k c_k^{N^f}(s,d)$ for network realizations N^f and $\min_k c_k^{N^g}(s,d)$ for network graph N^g. Therefore, N^f and N^g have the same cut values and the same maximum flows. ■

Next, we construct the wireless network codes using the linear network codes on the wired network graph N^g.

Theorem 4: There exist linear wireless network codes that can achieve the maximum flows for the wireless network realizations N^f, if there exist linear wired network codes that achieve the maximum flows on the wired network graph N^g.

Proof: We use a low complexity algorithm [8] to find linear network codes on N^g that achieve the maximum flow $\min_{d\in D}\min_k c_k^{N^g}(s,d)$ between source s and destinations $d\in D$. By Lemma 1, the value of the minimum cut on N^g is equal to the value of the maximum flow for the wireless network realizations N^f. We drop the time indices in the wireless network coding coefficients to obtain the notation for wired network coding. We assume the finite field F_2 for the network coding operations and construct the non-zero wireless network codes from the linear network codes on N^g as follows:

$$\alpha_{j,l,t}(tail(e)){=}1 \text{ if } \alpha_{j,e}{=}1, \quad t{>}l \text{ and } \exists m: e{\in}E^f_m,\ t{\in}\mathcal{T}_m$$

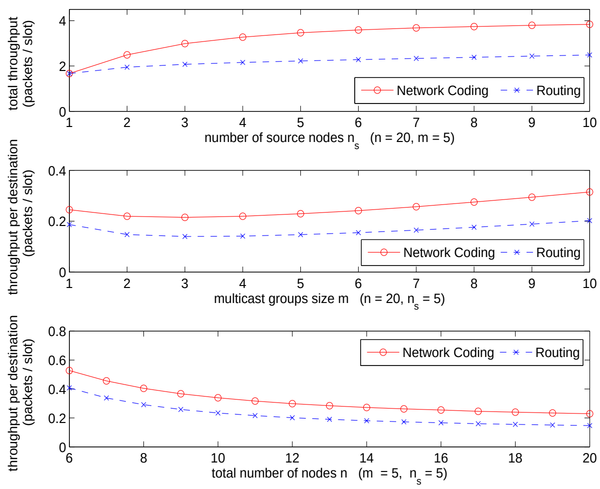

Fig. 5. Throughput comparison of network coding and routing

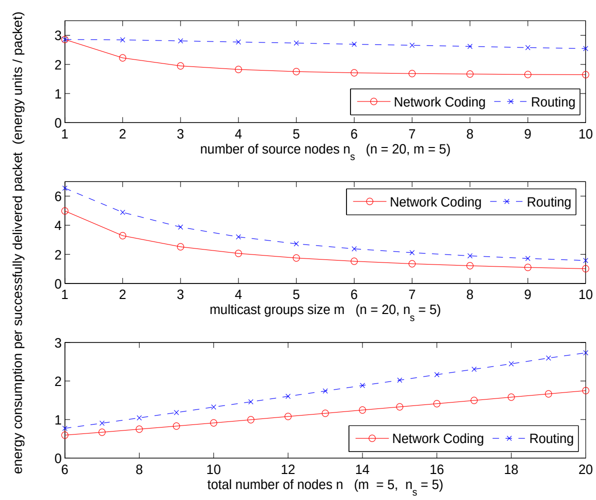

Fig. 6. Energy efficiency comparison of network coding and routing

$$\beta_{e,l,t}{=}1 \text{ if } \beta_{e,e'}{=}1, t{>}l \text{ and } \exists m,n\text{: } e'{\in}E^f_m, e{\in}E^f_n, t{\in}\mathcal{T}_m, l{\in}\mathcal{T}_n$$

$$\epsilon_{e,k,l,t}{=}1 \text{ if } \epsilon_{e,k}{=}1, \quad t{>}l \text{ and } \exists m: e{\in}E^f_m,\ l{\in}\mathcal{T}_m$$

These codes satisfy the wireless communication properties. Averaged over all network realizations N^f, the constructed wireless and wired network codes perform the same operations and achieve the same maximum flows for N^f and N^g. ■

Next, we determine wireless network codes for the network realizations in Figure 2. First, we construct the wired network graph N^g and find the non-zero linear network codes on N^g for the field F_2 as follows: $\alpha_{1,e_1} = \alpha_{2,e_2} = 1$ and $\beta_{e_1,e_7} = \beta_{e_1,e_3} = \beta_{e_2,e_4} = \beta_{e_2,e_8} = \beta_{e_3,e_5} = \beta_{e_3,e_6} = \beta_{e_4,e_5} = \beta_{e_4,e_6} = 1$. Then, we derive the non-zero wireless network codes using the method given in the proof of Theorem 4: (I) At time $\tau = 1$, $\alpha_{1,0,1}(s) = 1$, (II) At time $\tau \in \mathcal{T}_1$, $\beta_{e_1,l,\tau} = 1$, for $l = 1$ or $l = \tau{-}2 \in \mathcal{T}_2$, $\alpha_{2,\tau-1,\tau}(s) = 1$, $\epsilon_{e_7,1,\tau,\tau} = 1$, (III) At time $\tau \in \mathcal{T}_2$, $\beta_{e_2,\tau-1,\tau} = 1$, $\alpha_{1,\tau-1,\tau}(s) = 1$, $\epsilon_{e_8,2,\tau,\tau} = 1$, (IV) At time $\tau \in \mathcal{T}_3$, $\beta_{e_3,\tau-2,\tau} = 1$, $\beta_{e_4 \tau-1,\tau} = 1$, $\epsilon_{e_5,2,\tau,\tau} = 1$, $\epsilon_{e_7,2,\tau-2,\tau} = 1$, $\epsilon_{e_6,1,\tau,\tau} = 1$, $\epsilon_{e_8,1,\tau-1,\tau} = 1$.

VIII. Comparison of Network Coding and Routing

We consider tandem networks with at most two-neighbor connectivity and enumerate nodes from left to right in increasing order. Network coding in tandem networks consists of relay nodes adding the bits that arrive from left and right neighbors and sending the bit sum to both neighbors in single transmission. Network coding improves throughput and energy efficiency over routing, if and only if there exists a node i such that $x^{(m_1)}_{i-1,i}(d_1){>}0$ and $x^{(m_2)}_{i+1,i}(d_2){>}0$ for any pair of destinations $\{d_1{>}i,\ d_2{<}i\}$ and wireless network realizations $\{N^f_{m_1}, N^f_{m_2}\}$. Network coding cannot improve routing in tandem networks with single source or with directional transmissions.

Next, we address the problem of multicasting with multiple number of sources from a set S. We use the heuristic of section V-A to determine the network realizations, e.g. we have transmitter groups $T^{(m)}{=}\{3j{+}m, j{=}0,1,2,...\}$ for $m{=}1,2,3$ in tandem networks. We choose time fractions $\{\tau_i\}^M_{m=1}$ to maximize $\min_{s\in S}\min_{d\in D(s)}\min_k\{c^{N^f}_k(s,d)\}$, where $D(s)$ is the set of destinations for the source $s \in S$. For numerical results, we consider a tandem network with total of n nodes and n_s source nodes. Each source node independently and randomly chooses m destinations for its packets of one bit. We consider the classical collision channel model and assume unit energy cost for each transmission. For different values of n_s, m and n, Figures 5 and 6 depict the throughput and energy cost per packet, respectively. Numerical results show that network coding outperforms routing in terms of throughput and energy efficiency. We expect similar results for other topologies.

IX. Conclusions

In this paper, we extended network coding to wireless packet networks under the realistic assumptions such as omnidirectional transmissions, single transceiver per node and interference effects among concurrent transmissions. We proposed a joint scheduling and network coding solution that separately activates conflict-free network realizations with minimum cost assignments and optimizes the throughput or energy measures through network coding. We specified the properties of wireless network codes and outlined how to construct linear codes. Finally, we presented numerical results to compare throughput and energy efficiency of network coding and routing solutions.

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